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Hayashi *Econometrics*: Answers to Selected Review Questions

Chapter 6

Section 6.1

1. Let $s_n \equiv \sum_{j=1}^{n} |\gamma_j|$. Then $s_m - s_n = \sum_{j=n+1}^{m} |\gamma_j|$ for $m > n$. Since $|s_m - s_n| \to 0$, the sequence $\{s_n\}$ is Cauchy, and hence is convergent.

3. *Proof that* "$\beta(L) = \alpha(L)^{-1}\delta(L) \Rightarrow \alpha(L)\beta(L) = \delta(L)$":

$$\alpha(L)\beta(L) = \alpha(L)\alpha(L)^{-1}\delta(L) = \delta(L).$$

Proof that "$\alpha(L)\beta(L) = \delta(L) \Rightarrow \alpha(L) = \delta(L)\beta(L)^{-1}$":

$$\delta(L)\beta(L)^{-1} = \alpha(L)\beta(L)\beta(L)^{-1} = \alpha(L).$$

Proof that "$\alpha(L) = \delta(L)\beta(L)^{-1} \Rightarrow \alpha(L)\beta(L) = \delta(L)$":

$$\alpha(L)\beta(L) = \delta(L)\beta(L)^{-1}\beta(L) = \delta(L)\beta(L)\beta(L)^{-1} = \delta(L).$$

4. The absolute value of the roots is 4/3, which is greater than unity. So the stability condition is met.

Section 6.2

1. By the projection formula (2.9.7), $\widehat{\mathrm{E}}^*(y_t|1, y_{t-1}) = c + \phi y_{t-1}$. The projection coefficients does not depend on t. The projection is not necessarily equal to $\mathrm{E}(y_t|y_{t-1})$. $\widehat{\mathrm{E}}^*(y_t|1, y_{t-1}, y_{t-2}) = c + \phi y_{t-1}$. If $|\phi| > 1$, then y_{t-1} is no longer orthogonal to ε_t. So we no longer have $\widehat{\mathrm{E}}^*(y_t|1, y_{t-1}) = c + \phi y_{t-1}$.

3. If $\phi(1)$ were equal to 0, then $\phi(z) = 0$ has a unit root, which violates the stationarity condition. To prove (b) of Proposition 6.4, take the expected value of both sides of (6.2.6) to obtain

$$\mathrm{E}(y_t) - \phi_1 \mathrm{E}(y_{t-1}) - \cdots \phi_p \mathrm{E}(y_{t-p}) = c.$$

Since $\{y_t\}$ is covariance-stationary, $\mathrm{E}(y_t) = \cdots - \mathrm{E}(y_{t-p}) = \mu$. So $(1 - \phi_1 - \cdots - \phi_p)\mu = c$.

Section 6.3

4. The proof is the same as in the answer to Review Question 3 of Section 6.1, because for inverses we can still use the commutatibity that $\mathbf{A}(L)\mathbf{A}(L)^{-1} = \mathbf{A}(L)^{-1}\mathbf{A}(L)$.

5. Multiplying both sides of the equation in the hint from the left by $\mathbf{A}(L)^{-1}$, we obtain $\mathbf{B}(L)[\mathbf{A}(L)\mathbf{B}(L)]^{-1} = \mathbf{A}(L)^{-1}$. Multiplying both sides of this equation from the left by $\mathbf{B}(L)^{-1}$, we obtain $[\mathbf{A}(L)\mathbf{B}(L)]^{-1} = \mathbf{B}(L)^{-1}\mathbf{A}(L)^{-1}$.

Section 6.5

1. Let $\mathbf{y} \equiv (y_n, \ldots, y_1)'$. Then $\text{Var}(\sqrt{n}\overline{y}) = \text{Var}(\mathbf{1}'\mathbf{y}/n = \mathbf{1}' \text{Var}(\mathbf{y})\mathbf{1}/n)$. By covariance-stationarity, $\text{Var}(\mathbf{y}) = \text{Var}(y_t, \ldots, y_{t-n+1})$.

3. $\lim \gamma_j = 0$. So by Proposition 6.8, $\overline{y} \underset{m.s.}{\rightarrow} \mu$, which means that $\overline{y} \underset{p}{\rightarrow} \mu$.

Section 6.6

1. When $\mathbf{z}_t = \mathbf{x}_t$, the choice of $\mathbf{S}$ doesn't matter. The efficient GMM etimator reduces to OLS.

2. The etimator is consistent because it is a GMM etimator. It is not efficient, though.

Section 6.7

2. $J = \widehat{\boldsymbol{\varepsilon}}'\mathbf{X}(\mathbf{X}'\widehat{\boldsymbol{\Omega}}\mathbf{X})^{-1}\mathbf{X}'\widehat{\varepsilon}$, where $\widehat{\boldsymbol{\varepsilon}}$ is the vector of estimated residuals.

4. Let $\widehat{\omega}_{ij}$ be the (i, j) element of $\widehat{\boldsymbol{\Omega}}$. The truncated kernel-based estimator with a bandwidth of q can be written as (6.7.5) with $\widehat{\omega}_{ij} = \widehat{\varepsilon}_i\widehat{\varepsilon}_j$ for (i, j) such that $|i - j| \leq q$ and $\widehat{\omega}_{ij} = 0$ otherwise. The Bartlett kernel based estimator obtains if we set $\widehat{\omega}_{ij} = \frac{q-|i-j|}{q}\widehat{\varepsilon}_i\widehat{\varepsilon}_j$ for (i, j) such that $|i - j| < q$ and $\widehat{\omega}_{ij} = 0$ otherwise.

5. $\text{Avar}(\widehat{\beta}_{\text{OLS}}) > \text{Avar}(\widehat{\beta}_{\text{GLS}})$ when, for example, $\rho_j = \phi^j$. This is consistent with the fact that OLS is efficient, because the orthogonality conditions exploited by GLS are different from those exploited by OLS.