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Hayashi *Econometrics*: Answers to Selected Review Questions

Chapter 5

Section 5.1

2. $\mathbf{b}_i = (1, IQ_i)'$, $\boldsymbol{\beta} = (\phi_2 - \phi_1, \phi_3 - \phi_1, \beta)'$, and $\boldsymbol{\gamma} = (\phi_1, \gamma)'$.

3. Let $\mathbf{s}_i$ be $(S69, S80, S82)'$. Then $\mathbf{QF}_i = [\mathbf{Q} \vdots \mathbf{Qs}_i]$. So $\mathbf{QF}_i \otimes \mathbf{x}_i = [\mathbf{Q} \otimes \mathbf{x}_i \vdots \mathbf{Qs}_i \otimes \mathbf{x}_i]$ and

$$\underset{(3K\times 4)}{\mathrm{E}(\mathbf{QF}_i \otimes \mathbf{x}_i)} = [\underset{(3K\times 3)}{\mathrm{E}(\mathbf{Q} \otimes \mathbf{x}_i)} \vdots \underset{(3K\times 1)}{\mathrm{E}(\mathbf{Qs}_i \otimes \mathbf{x}_i)}]$$

$$\mathrm{E}(\mathbf{Q} \otimes \mathbf{x}_i) = \begin{bmatrix} 2/3\,\mathrm{E}(\mathbf{x}_i) & -1/3\,\mathrm{E}(\mathbf{x}_i) & -1/3\,\mathrm{E}(\mathbf{x}_i) \\ -1/3\,\mathrm{E}(\mathbf{x}_i) & 2/3\,\mathrm{E}(\mathbf{x}_i) & -1/3\,\mathrm{E}(\mathbf{x}_i) \\ -1/3\,\mathrm{E}(\mathbf{x}_i) & -1/3\,\mathrm{E}(\mathbf{x}_i) & 2/3\,\mathrm{E}(\mathbf{x}_i) \end{bmatrix}.$$

The columns of this matrix are not linearly independent because they add up to a zero vector. Therefore, $\underset{(3K\times 4)}{\mathrm{E}(\mathbf{QF}_i \otimes \mathbf{x}_i)}$ cannot be of full column rank.

Section 5.2

1. No.

4. Since $\widetilde{\boldsymbol{\eta}}_i = \mathbf{Q}\boldsymbol{\varepsilon}_i$, $\mathrm{E}(\widetilde{\boldsymbol{\eta}}_i \widetilde{\boldsymbol{\eta}}_i') = \mathbf{Q\Sigma Q}$, where $\boldsymbol{\Sigma} \equiv \mathrm{E}(\boldsymbol{\varepsilon}_i \boldsymbol{\varepsilon}_i')$. This matrix cannot be nonsingular, because $\mathbf{Q}$ is singular.

Section 5.3

1.

$$\mathbf{Q} = \begin{bmatrix} 1/2 & 0 & -1/2 \\ 0 & 1 & 0 \\ -1/2 & 0 & 1/2 \end{bmatrix}.$$

Section 5.4

2(b) If $\mathrm{Cov}(s_{im}, y_{im} - y_{i,m-1}) = 0$ for all m, then $\boldsymbol{\Sigma}_{\mathbf{xz}}$ becomes

$$\boldsymbol{\Sigma}_{\mathbf{xz}} = \begin{bmatrix} 1 & 0 & \mathrm{E}(y_{i1} - y_{i0}) \\ \mathrm{E}(s_{i1}) & 0 & \mathrm{E}(s_{i1})\,\mathrm{E}(y_{i1} - y_{i0}) \\ 0 & 1 & \mathrm{E}(y_{i2} - y_{i1}) \\ 0 & \mathrm{E}(s_{i2}) & \mathrm{E}(s_{i2})\,\mathrm{E}(y_{i2} - y_{i1}) \end{bmatrix}.$$

This is not of full column rank because multiplication of $\boldsymbol{\Sigma}_{\mathbf{xz}}$ from the right by $(\mathrm{E}(y_{i1} - y_{i0}), \mathrm{E}(y_{i2} - y_{i1}), 1)'$ produces a zero vector.