How to Go from Cointegrated VAR to VMA?

On p. 637, we stated that a VAR (10.2.12) is cointegrated of rank h if and only if $\mathbf{\Phi}(L)$ can be factored as $\mathbf{\Phi}(L) = \mathbf{U}(L)\mathbf{M}(L)\mathbf{V}(L)$, where $\mathbf{U}(L)$ and $\mathbf{V}(L)$ are $n \times n$ matrix lag polynomials with all their roots outside the unit circle and $\mathbf{M}(L)$ is a matrix polynomial given by

$$\mathbf{M}(L) \equiv \begin{bmatrix} (1-L)\mathbf{I}_{n-h} & \underset{((n-h)\times h)}{\mathbf{0}} \\ \underset{(h\times(n-h))}{\mathbf{0}} & \mathbf{I}_h \end{bmatrix}.$$

It is true that the VMA representation $\Delta\boldsymbol{\xi}_t = \mathbf{\Psi}(L)\boldsymbol{\varepsilon}_t$ can be computed from $\mathbf{\Phi}(L)$ by (10.2.17), but there is a way to write $\mathbf{\Psi}(L)$ explicitly that exploits the above factorization. The derivation is as follows (I suppose the result to be shown below is due to Sam Yoo, but I follow Watson's (1994, pp. 2872-73) exposition). The VAR representation under the factorization is

$$\mathbf{U}(L)\mathbf{M}(L)\mathbf{V}(L)\boldsymbol{\xi}_t = \boldsymbol{\varepsilon}_t.$$

Multiply both sides from left by $\mathbf{U}(L)^{-1}$ to obtain

$$\mathbf{M}(L)\mathbf{V}(L)\boldsymbol{\xi}_t = \mathbf{U}(L)^{-1}\boldsymbol{\varepsilon}_t. \qquad (*)$$

(Both sides are well-defined because $\mathbf{U}(L)^{-1}$ is absolutely summable by the vector version of Proposition 6.3.) Now define

$$\overline{\mathbf{M}}(L) \equiv \begin{bmatrix} \mathbf{I}_{n-h} & \underset{((n-h)\times h)}{\mathbf{0}} \\ \underset{(h\times(n-h))}{\mathbf{0}} & (1-L)\mathbf{I}_h \end{bmatrix}.$$

Multiplying both sides of $(*)$ from left by $\overline{\mathbf{M}}(L)$ and noting that $\overline{\mathbf{M}}(L)\mathbf{M}(L) = (1-L)\mathbf{I}_n$ and $(1-L)\mathbf{V}(L) = \mathbf{V}(L)(1-L)$, we obtain

$$\mathbf{V}(L)\Delta\boldsymbol{\xi}_t = \overline{\mathbf{M}}(L)\mathbf{U}(L)^{-1}\boldsymbol{\varepsilon}_t.$$

Multiply both sides of this from left by $\mathbf{V}(L)^{-1}$, which is absolutely summable, we obtain

$$\Delta\boldsymbol{\xi}_t = \mathbf{\Psi}(L)\boldsymbol{\varepsilon}_t, \quad \text{with} \quad \mathbf{\Psi}(L) \equiv \mathbf{V}(L)^{-1}\overline{\mathbf{M}}(L)\mathbf{U}(L)^{-1}.$$