TABLE 4

RETURNS TO SCALE AND ELASTICITIES OF OUTPUT WITH RESPECT TO VARIOUS INPUTS DERIVED FROM RESULTS PRESENTED IN TABLE 3 FOR 145 FIRMS IN 1955

Regression No.	Returns to Scale	Elasticity of Output with Respect to		
		Labor	Capital	Fuel
I	1.39	0.78	−0.00	0.61
II	1.44	0.74	0.01	0.69
IIIA	2.52	1.61	−0.02	0.93
IIIB	1.50	0.16	0.53	0.81
IIIC	1.08	0.44	0.27	0.37
IIID	1.09	0.52	0.15	0.42
IIIE	0.96	0.58	−0.29	0.67
IVA	2.52	1.10	0.25	1.17
IVB	1.53	0.65	0.15	0.73
IVC	1.14	0.50	0.11	0.53
IVD	1.10	0.48	0.11	0.51
IVE	0.94	0.41	0.09	0.44

The difficulties with capital may be due in part to the difficulty I encountered in measuring both capital costs and the price of capital. The former were measured as depreciation charges plus the proportion of interest on long-term debt attributable to the production plant; the figure for capital price was compounded of the yield on the firm's long-term debt and an index of construction costs. Depreciation figures reflect past prices and purchases of capital equipment, whereas the price of capital as I constructed it does not; it is perhaps not so surprising then that the price has little effect on costs. Model B is designed to evade this difficulty. Results based on Model B are presented in line V of Table 5 and the implications of this regression for the parameters in the production function are given in line V of Table 6. It is apparent that the estimates of returns to scale and the elasticities of output with respect to labor and fuel are changed very little;

TABLE 5

RESULTS FROM REGRESSIONS BASED ON MODEL B FOR 145 FIRMS IN 1955. DEPENDENT VARIABLE WAS C = LOG COSTS

Regression No.	Coefficient			R^2
	Y	P_1	P_3	
V	0.723 (±.019)	0.483 (±.303)	0.496 (±.106)	0.914
VIA	0.361 (±.086)	0.212 (±1.259)	0.655 (±.350)	0.438
VIB	0.661 (±.106)	−0.401 (±.333)	0.490 (±.134)	0.672
VIC	0.985 (±.180)	−0.014 (±.261)	0.330 (±.138)	0.647
VID	0.927 (±.106)	0.327 (±.228)	0.426 (±.064)	0.884
VIE	1.035 (±.067)	0.704 (±.272)	0.643 (±.132)	0.934

Figures in parentheses are the standard errors of the coefficients.

TABLE 6

RETURNS TO SCALE AND ELASTICITIES OF OUTPUT WITH RESPECT TO VARIOUS INPUTS DERIVED FROM RESULTS PRESENTED IN TABLE 5 FOR 145 FIRMS IN 1955.

Regression No.	Returns to Scale	Elasticity of Output with Respect to		
		Labor	Capital	Fuel
V	1.38	0.67	0.03	0.69
VIA	2.77	0.59	1.39	0.74
VIB	1.51	−0.62	0.69	0.33
VIC	1.02	−0.01	0.27	0.46
VID	1.08	0.35	−0.34	0.62
VIE	0.97	0.68	0.03	0.68

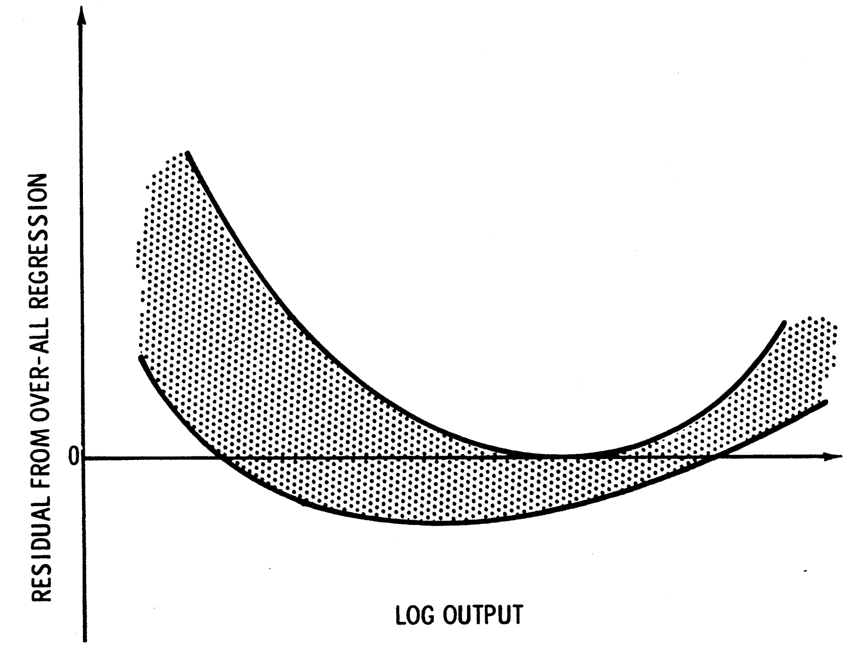

FIG. 1

the elasticity with respect to capital is of the right sign but still unreasonably low for an industry that is so capital-intensive.[4]

A second difficulty with these regressions is not apparent from an examination of the coefficients and their standard errors. As part of these analyses, the residuals from the regressions were plotted against the logarithm of output. The result is schematically pictured in Fig. 1. It is clear that neither regression relationship is truly linear in logarithms. To test this visual impression the observations were arranged in order of ascending output, and Durbin-Watson statistics were computed; the values of the statistics indicated highly significant positive serial correlation, which confirmed the visual evidence.

Aside from difficulties with the basic data, there appear to be at least two plausible and interesting hypotheses accounting for the result.

[4] K. Arrow has pointed out that considerations of plausibility implicitly involve an alternative method of estimating the coefficients in the production function: From the marginal productivity conditions (3), we find that for any pair of inputs i and j,

$$\frac{p_i x_i}{p_j x_j} = \frac{a_i}{a_j}.$$

Hence, by constructing some average of the ratios of expenditures on factors, we obtain estimates of the ratios of exponents in the production function. Had the data been arranged in such a manner as to facilitate computation of expenditures on individual factors, a comparison of the ratios a_i/a_j obtained in this way with those derived from the cost function would have been a useful supplement to the analysis. Arrow also pointed out that one could also verify the results by the fit of the production function derived from them. Unfortunately, it is not feasible to obtain good physical measures of the inputs, and such measures are required for this test.

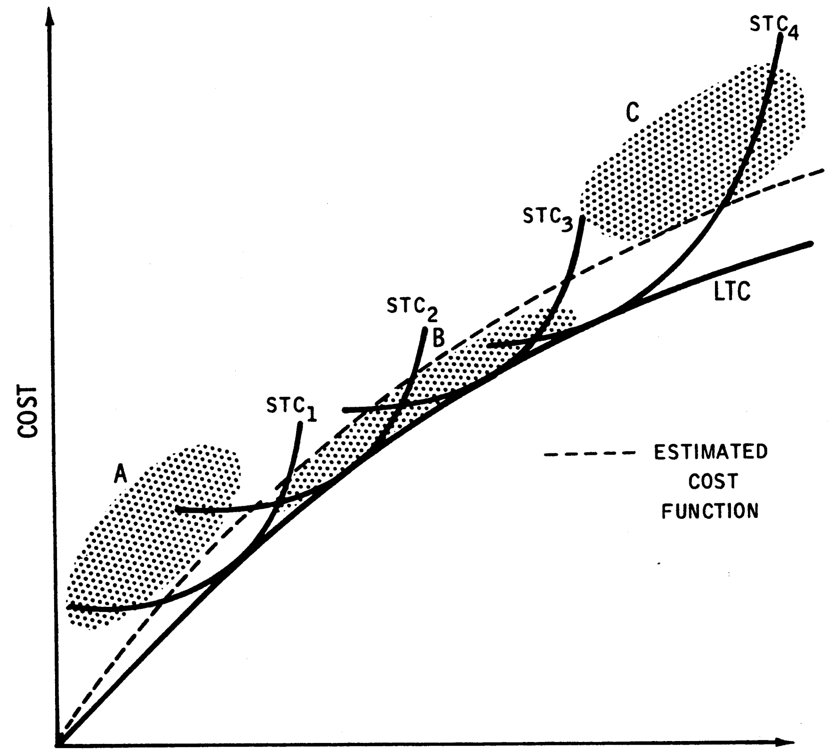

FIG. 2

1. The first explanation of the result derives from dynamic considerations closely related to those underlying Friedman's Permanent-Income Hypothesis [7]. The important thing to note is that actual costs are underestimated by the regressions at both high and low outputs. Consider the situation pictured in Fig. 2. Firms operate not on the long-run cost curve, but at points on the various short-run curves. If firms are evenly distributed about their optimal outputs (i.e., outputs at which long-run marginal cost equals short-run marginal cost), the effect will be to increase the estimate of the extent of increasing returns to scale if they are increasing, or diminish further the estimate of returns to scale if they are decreasing.[5] But elsewhere Friedman holds that a uniform distribution is not likely to occur; in fact he says, "The firms with the largest output are unlikely to be producing at an unusually low level: on the average they are likely to be producing at an unusually high level; and conversely for those that have the lowest output" [14, p. 237].

The situation described by Friedman is pictured in Fig. 2 by the shaded areas A, B, and C, which refer, respectively, to observations on firms with unusually low, usual, and unusually high outputs. The Friedman explana-

[5] This argument rests partly on the form of the function that constrains it to pass through the origin.

tion does produce a residual pattern similar to that observed. Regression II, Table 3, is designed to test this explanation for Model A. A corresponding test for Model B was not made. Since "usual" output cannot be directly observed, the hypothesis was modified slightly by identifying departure from the usual with large changes in output from the previous year, the assumption being that firms with stable output were likely to be near the optimal long-run output.[6] Thus, the absolute percentage changes in output should be positively related to total costs. Unfortunately, they are negatively related and significantly so.

Part of the explanation for this unexpected result is suggested by a more careful examination of the data. Almost all firms with large changes had positive changes and had been experiencing rapid growth for some time. It is well known, though unfortunately not taken into account in these analyses, that there is a steady rate of technological progress in generating equipment. Since expanding firms purchase new equipment in the process, the average age of a plant in those firms experiencing large changes in output is lower than that of firms with more stable outputs. Hence, the former tend to have lower costs because of the inadequacy of the capital-cost data to reflect obsolescence.[7] Thus, while one would not want to reject the Friedman hypothesis on the basis of this evidence, it clearly does not explain the residual pattern.

2. Fortunately, the observed result can be explained by a much simpler hypothesis, namely, that the degree of returns to scale is not independent of output, but varies inversely with it. Figure 3 illustrates this explanation: The solid line gives the traditional form of the total cost function, which shows increasing returns at low outputs and decreasing returns at high outputs. If we try to fit a function for which returns to scale are independent of the level of output, e.g., one linear in logarithms, a curve such as the dashed one will be obtained. The shaded areas A and B show the output ranges, high and low, for which total costs are underestimated.

[6] Capacity figures might have been used. However, those available appear to be somewhat unrealistic. These are based on generator name-plate ratings, which refer to the maximum output that can be produced without overheating. According to the Federal Power Commission, however, units of the same size, general design, and actual capability may show as much as a 20 per cent difference in rating [5, p. xi]. Furthermore, in a multiple-plant firm, total generator capacity is not the only factor to be considered. Such defects in the capacity figures also led to grouping firms by output rather than by capacity in the analyses of covariance presented below.

[7] Treatment of capital costs is the source of one of the most serious shortcomings of the present study, as indeed capital measurement is in most studies of production. Solow's recent contribution to the study of the aggregate production function [18] offers considerable promise of an appropriate measure of capital used in the production of electric power. I hope, in future work, to make use of a model of production that involves fixed coefficients *ex post* at the plant level, but that permits substitution of inputs and that changes over time *ex ante*.